MODELING DISTRIBUTIONS OF STEM CHARACTERISTICS
OF GENETICALLY IMPROVED LOBLOLLY PINE

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Abstract.--Diameter distribution models for genetically improved and unimproved loblolly pine were developed using the moment-based beta probability density function. The parameters of the models were based on stand age, stand density, and average height of the dominants. Comparisons were made between the improved and unimproved predicted distributions to determine the effect of genetic improvement on stand growth and yield. Preliminary results indicate that diameter distributions of improved females are less peaked and are shifted toward larger diameter classes than the unimproved females.

Additional keywords: *Pinus taeda*, tree improvement, diameter distributions, growth and yield.

The advancement of tree improvement, that is, increasing productivity with the use of genetically improved stock, has made considerable gains in improving individual tree characteristics. However, knowledge regarding the effect of genetic variation on stand growth and yield at harvest age is limited. Yield tables that are currently available apply only to natural stands or plantations established from "woodsrun" stock. It is essential that existing growth and yield models be modified to incorporate the effects of genetic improvement if reliable estimates of genetic improvement at a stand level are to be made.

In recent years, experiments established in the past have reached sufficient age for the long term effects of genetic improvement to be tested. For this study, individual tree data collected from half-sib family-block plantings of rotation-aged stands of genetically improved loblolly pine (*Pinus taeda* L.) established by the Texas Forest Service were used to model diameter distributions over time. These distributions were then compared to diameter distributions of unimproved loblolly pine grown in the same plantings. Quantification of the similarities and differences in the shapes and levels of the distributions through time for the genetically improved and unimproved loblolly pine provided additional understanding of the development of stands of improved stock and information for modeling their growth.

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DATA

Three loblolly pine plantations (001, 006, 041), in which the trees were planted in a replicated randomized block design, were included in the research. Plantations 001 and 041 were established in 1952-53 and 1955-56, respectively, in Many, LA at the A. J. Hodges Experiment Area. The measurement plots were composed of the inner 64 trees of 100 trees per plot planted in a 7 by 7 foot spacing. In plantation 001, there were twelve females, comprising six improved females and six unimproved females, and in plantation 041 there were four females, two improved and two unimproved females. Plantation 006 was established in 1956-57 in Fastrill, TX at the Arthur H. Temple Research Area, with thirty females, fifteen improved and fifteen unimproved. In each of the plantations, the mother trees for the seed sources were found in the same proximity; the improved and unimproved females were then planted side by side for paired comparisons.

The measurements taken consisted of individual tree diameter outside bark at breast height and total height at ages 1, 2, 3, 5, 10, 15, 20, and 30 from planting. Plantation 001 was thinned at ages 10 and 17, plantation 006 at age 13, and plantation 041 at age 14.

METHOD OF ANALYSIS

The frequency distributions of diameter measurements have been described using probability density functions (pdf). An essential feature in using pdf's is the ability to predict the pdf parameter values for a given set of stand conditions. In the past many different probability distributions have been used. For this study, the beta distribution was chosen since it can assume a variety of shapes, can be easily fit by moments, and has been utilized successfully in several previous growth and yield studies. Yield tables based on the beta distribution have been prepared by Bennett and Clutter (1968), Beck and Della-Bianca (1970), Lenhart and Clutter (1971), and Lenhart (1972).

The beta distribution, as presented by Krutchkoff (1970), is of the form:

$$f(x) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} \quad x^{\alpha-1}\,(1-x)^{\beta-1} & 0<x<1 \text{ and } \alpha,\beta>0 \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

Since the family of beta distributions is a family of probability densities of continuous random variables taking on values in the interval (0,1), the previous formula must be modified to account for diameter data in the interval (Dmin, Dmax). The following equation is used to code the tree diameters (Di) so that all diameters in a plot fall in the range of the beta distribution (O<Xi<1):

$$Xi = \frac{Di - Dmin}{Dmax - Dmin} \tag{2}$$

where,

X_i — scaled tree diameter
D_i — unscaled tree diameter
Dmin — minimum tree diameter observed on a particular plot
Dmax — maximum tree diameter observed on a particular plot

Thus, the following modification of equation (1) results:

$$f(D) = \begin{cases} \left[\dfrac{\Gamma(\alpha+\beta)}{(Dmax - Dmin)\Gamma(\alpha)\Gamma(\beta)}\right] \left[\dfrac{D - Dmin}{Dmax - Dmin}\right]^{\alpha-1} \left[\dfrac{D - Dmin}{Dmax - Dmin}\right]^{\beta-1} \\ 0 \text{ otherwise} \end{cases} \quad (3)$$

$$Dmin<D<Dmax;\ \alpha,\beta>0$$

where,

f(D) — the relative frequency of occurrence of a given diameter, D
m1/3 — model parameters to be estimated from the data
Dmin, Dmax as above

To determine the parameters of the beta distribution, a and p, the method of moments was used instead of the method of maximum likelihood. This was because moment estimates are easier to compute and both Lloyd (1966) and Strub (1972) found that relatively small differences existed between the parameter estimates of the two techniques. The method of moment equations were computed from data grouped into 1-centimeter class intervals. The first and second noncentral moments of x (or diameter) were estimated by the average diameter of the stand and the basal area per acre. Once the parameter estimates for the beta distribution were made on the 56 plots, the predicted diameter distribution was compared to the observed diameter distribution by means of the X^2 Goodness-of-Fit test.

The moment estimates of the parameters a and p were predicted for each plot using stand age, average height of the dominant trees, number of trees per hectare, and genetic information. Average height of the dominant trees was used in place of site index since height was a measured variable. Average height of the dominant trees for the unimproved females was also tested. The genetic information was included by means of coding each of the 20 different females for the three plantations combined as genetically improved or unimproved. This was done by means of indicator variables. Each group of a genetically improved (superior) female and unimproved (check) female was testes independently of the other groups. Numerous models were tested using different combinations of the stand conditions, including Burkhart and Strub's (1974) parameter prediction equations for estimating a and .3 of the beta pdf. Dmin and Dmax were also predicted by multiple regression techniques using the same independent variables of stand age, average height of the dominant trees, and number of trees per hectare.

The regression models were analyzed using fit and prediction statistics. Fit statistics, such as the F-values, the coefficient of determination (R^2), the sum of square error (SSE), and the mean squared error (MSE), were used to determine how well the data conformed to the model. Prediction statistics, such as PRESS, Cp, and VIF, give an indication of a model's predictive ability. The PRESS (Predictive Error Sum of Squares) evaluates alternative models when the objective is prediction. The model which yields the lowest PRESS value may predict better than the other models. The Cp criterion evaluates the bias of the regression model, and the variance inflation factor (VIF) indicates if there is multicollinearity. Residuals, the deviation between data and fit, were plotted to detect any other model inadequacies.

RESULTS AND DISCUSSION

The following results are given for one grouping of a superior family and a related check family. Similar results were found for the other groups. Comparisons of the average diameter at breast height and the average height of the dominants at each age for the superior and check families are shown (Figures 1, 2a). Comparisons of the number of trees per hectare, the basal area per hectare, and volume per hectare for the same group are also shown (Figure 2b, c, and d).

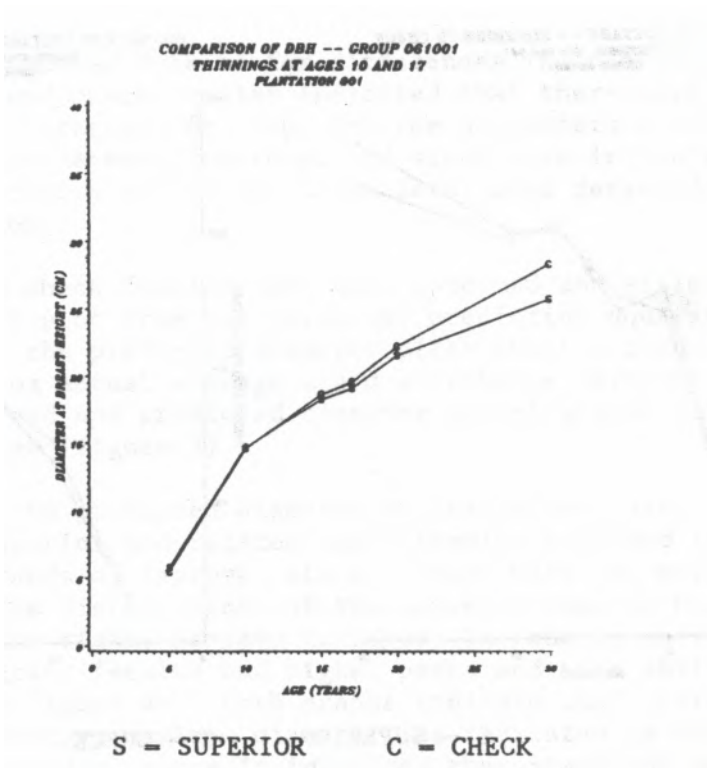

Figure 1. Comparison of the superior versus check female for average diameter at breast height.

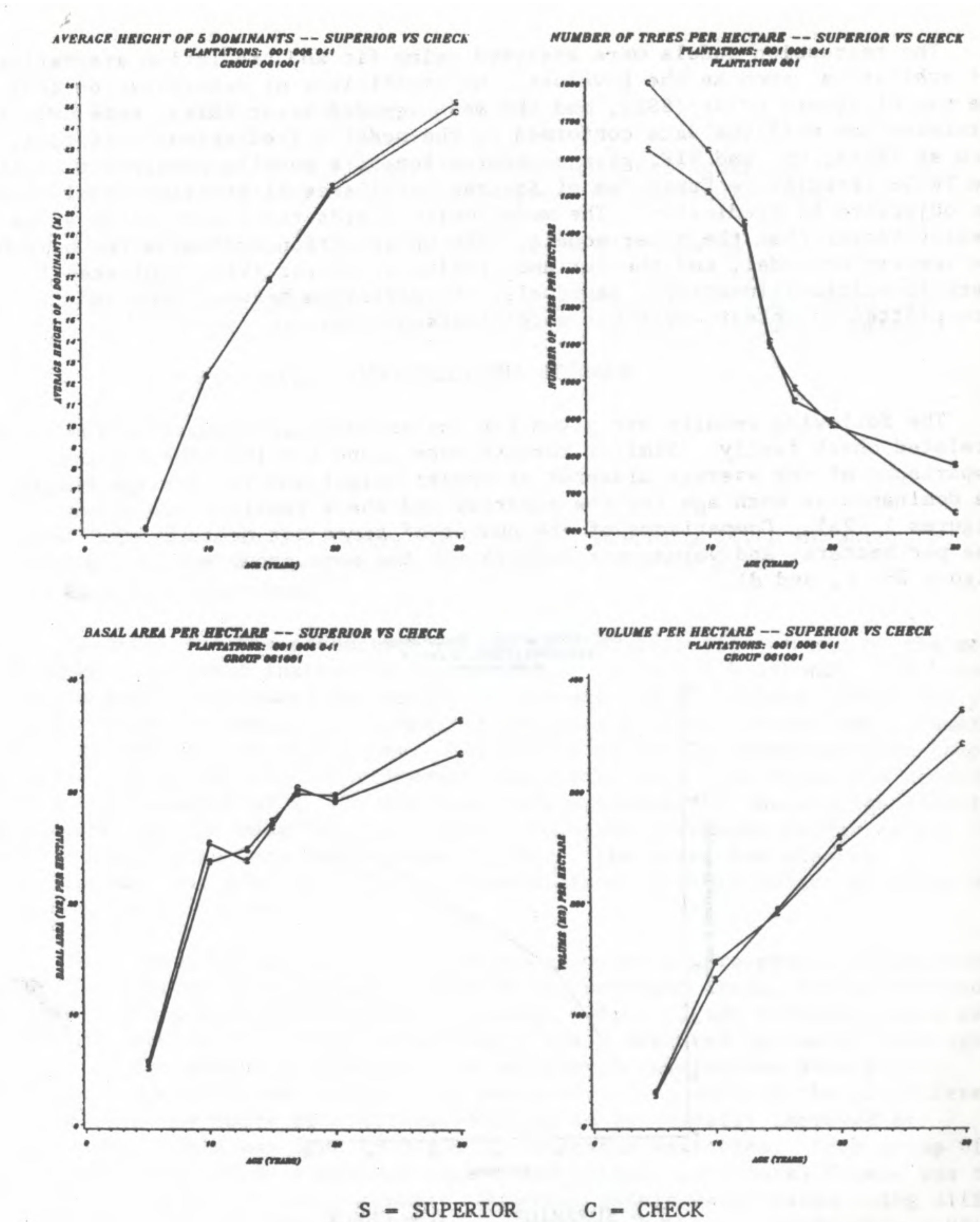

Figure 2. Comparison of the superior versus check female for a) average height of the dominants, b) number of trees per hectare, c) basal area per hectare, and d) volume per hectare.

Observed values of Dmin, Dmax, average diameter, and average squared diameter were used to solve for the parameters, a and 0, of the moment-based beta distribution. A solution was achieved for each of the 56 plots. Using the stand conditions, age, height of the dominants of the unimproved females (Chtd), and trees per hectare (Tph) from each sample plot, stand average attributes, Dmin, Dmax, average diameter, and average squared diameter, were predicted by linear regression equations and parameters recovered.

The models to predict the parameter estimates were chosen on the basis of the fit and prediction statistics as described in the method of analysis. These models were the same as the models determined by Burkhart and Strub (1974). They are:

$$\hat{\alpha} = \alpha_0 + \alpha_1 \text{(Age/Tph)} + \alpha_2 \text{(Age)(Chtd)}$$

$$\hat{\beta} = \alpha_0 + \alpha_1 \text{(Age/Tph)} + \alpha_2 \text{(Tph)(Chtd)}$$

$$\widehat{\text{Dmin}} = \alpha_0 + \alpha_1 \text{(Chtd)} + \alpha_2 \text{(Age)(Tph)} + \alpha_3 \text{(Chtd/Tph)}$$

$$\widehat{\text{Dmax}} = \alpha_0 + \alpha_1 \text{(Chtd)} + \alpha_2 \text{(Age)(Tph)} + \alpha_3 \text{(Chtd/Tph)}$$

The full versus reduced F-tests for differences in intercept and/or slope between the superior and check females indicated that there were no significant differences in either intercept or slope for the parameters and p. No significant differences between intercept and slope were indicated for Dmin, but significant differences at the .05 alpha level were detected for Dmax in both intercept and slope.

The superior and check females were then combined and estimates of and 0 were obtained for each plot from the parameter prediction equations. Using the estimates of cc and 0, the predicted diameter distributions produced by the moment-based beta using actual average stand attributes were determined for each plot. The observed and predicted diameter distributions using stand attributes were similar (Figure 3).

A comparison of the predicted diameter distributions using stand statistics for the superior and related check females provided information on the development of stands of improved stock. There were two main cases resulting: 1) that the distributions of the superior females had lower peaks and were shifted to the right, as seen in Figure 4a, and 2) that the distributions of superior females had higher peaks and were shifted slightly to the right, as seen in Figure 4b. Both graphs indicate that there is definitely a difference in the average maximum diameter, as indicated by the F-tests. This is an important result, since it indicates that stands of genetically improved females may have distributions with trees of greater diameters.

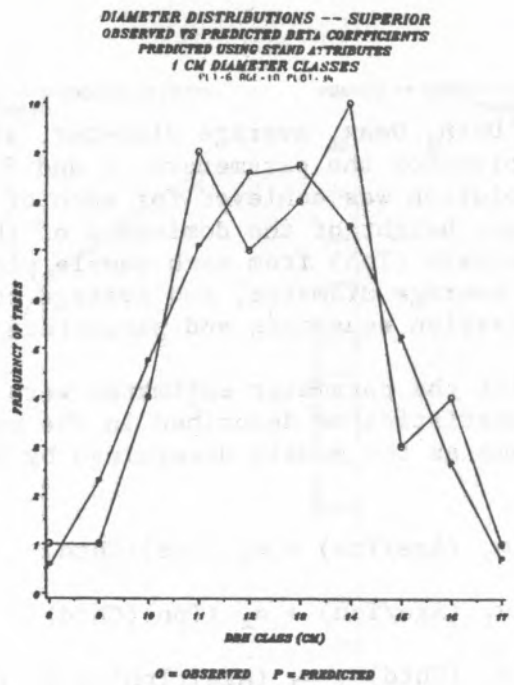

Figure 3. The observed versus the predicted diameter distribution using stand average attributes for a particular plot.

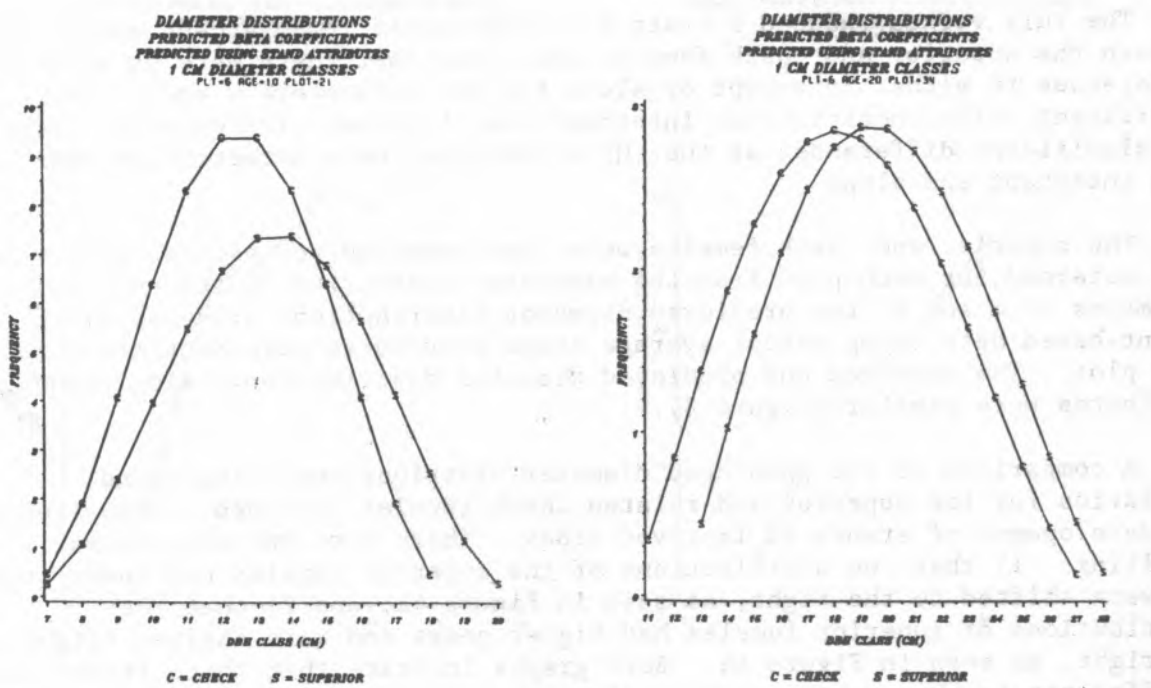

Figure 4. Comparisons of the predicted diameter distribution using stand attributes for a superior versus check female, where a) shows the first relationship and b) shows the second relationship..

CONCLUSIONS

The effect of genetic improvement on the development of diameter distributions of loblolly pine at harvest age has been tested for a limited sample of genetically improved and unimproved females. The comparisons in Figure 4 indicated that there are differences in the peaks and shapes of the distributions. The average maximum diameter for the improved females was found to be larger. These results are preliminary. Further models and comparisons gill be tested. In addition, two different probability density functions will be used, the Weibull and Johnson's Sb. The Weibull function has been utilized in several recent yield studies and has also been successful in quantifying diameter distributions. Johnson's Sb has not been used as extensively in growth and yield studies, but was found to have desireable characteristics in comparison to other distributions (Hafley and Schreuder 1977).

Comparing the results of the three different distributions will more accurately indicate the differences between the genetically improved and unimproved loblolly pine diameter distributions. Additional quantification of the effects of genetic improvement of the diameter distributions through time will provide increased understanding of the development of stands of improved stock and information for modeling their growth.

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