AREA POTENTIALLY AVAILABLE TO A TREE: A RESEARCH TOOL

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Abstract.--Accounting for competiton influences on individual tree growth has been a stumbling block for many forest scientists. Partitioning the forest floor into areas potentially available (APA) to individual trees (Brown 1965) provides a useful competition index that has been overlooked or avoided. Two reasons for this are a lack of understanding of this geometric construct and its properties and inefficient methods of computation. Advances in computational geometry and forestry have made APA and similar geometric constructs practical and useful research tools in the assessment of competition. The creation and properties of these geometric models are discussed.

Additional keywords: polygon, triangulation, competition index.

INTRODUCTION AND BACKGROUND

Analysis of individual tree data is extremely difficult due to the confounding of competition with other influences. Numerous competition indices exist for both the average competition in the stand and the individual tree. The performance of many of these is limited because spatial information is not incorporated in the index. The area potentially available (APA) uses the spatial information of a tree's competitors to develop an index to account for the effects of local density on a tree's growth.

One of the earliest writings on the concept of APA was by the German forester Koenig in 1864. Though this concept has been in existence for over a century, only small advances in the application and utility of APA have been made. One reason for this is that many forest scientists are unacquainted with APA as a research tool. For those acquainted with APA, the reasons it is not used are its intractability and lack of flexibility for different applications.

The background and mathematical origins and properties of APA are presented herein. Advances in computational geometry, computer science, and forestry are used to develop APA so it can be easily used by forest scientists.

The shapes most frequently used in the past to define APA have been circles and squares (Opie 1968; Gerrard 1969; Bella 1971; Smaltschinski 1981). Brown (1965) was the first to use a polygon to represent APA and introduced the term. Jack (1967) independently developed APA for use in individual tree sampling. Fraser and Van Den Driessche (1972) were the first in forestry to express the mathematical origins of APA

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and used the "least diagonal neighbor" network for the estimation of stand density and spatial distribution. Fraser (1977) went on to adapt this tessellation, partitioning of the plane, for use in a method of unequal probability individual tree sampling that could be easily implemented in the field. Moore et. al., (1973) compared weighted APA to Bella's competitive influence-zone overlap (CIO) competition index. APA performed as well if not better than CIO. Moore also noted that APA when weighted will result in areas that are not assigned to a tree. Pelz (1978) used APA to predict individual tree growth. Johnson and Pelz (1979) investigated the use of APA and weighted APA for forest inventory. Daniels (1981) compared APA and weighted APA to other competition indices for the prediction of Basal area growth. Weighted APA performed the best when other significant variables were added for the prediction of basal area growth.

TRIANGULATIONS

APA is based upon a network of triangles. The primary function of the triangulation used for APA is to define competitors and form a geometric construct from which area can be assigned to an individual. A tree is a competitor if it is close to the subject tree, but not blocked from competing by a closer tree. One way of approximating this is to create a triangulation in which the sum of the length of the edges is the minimum. This triangulation is the minimum weight triangulation (MWT). No algorithm will consistently create the MWT. The two best heuristics for the MWT are the Delaunay triangulation (DT) and Greedy triangulation (GT) (Lloyd 1977; Gilbert 1976; Manacher and Zobrist 1979).

The basis of Brown's APA is the DT (figure 1). The definition of a DT is that all triangles form a circumcircle that contain no other points in the plane. The triangle network used by Fraser was the least diagonal neighbor network, better known as the GT. The GT is generated by creating all possible edges of a set of points. Then all edges that do not intersect a shorter edge are in the tessellation. One important aspect of both triangulations is that they are unique.[2] How well these triangulations approximate the MWT varies. The sum of the length of the edges of the DT can be $n(N/\log N)$ as large as the MWT and the GT can be $8(N^{1/3})$ times greater than the MWT (Manacher and Zobrist 1979).[3]

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[2] The GT will not be unique in the rare event two or more equilength edges intersect.

[3] This notation was developed by D. E. Knuth (1976) for the analysis of algorithms. The notation '$O(f(N))$' means of the same or lesser order of f(N). '$\theta(f(N))$' means in exactly the same order as f(N) and '$n(f(N))$' means in the same or greater order of f(N). This notation is generally used in reference to the speed and memory of an algorithm, though it may refer to other quantities. When referring to speed f(N) is the number of primary operations an algorithm must perform based on the number, N, of datum input. This notation with respect to memory indicates the amount needed.

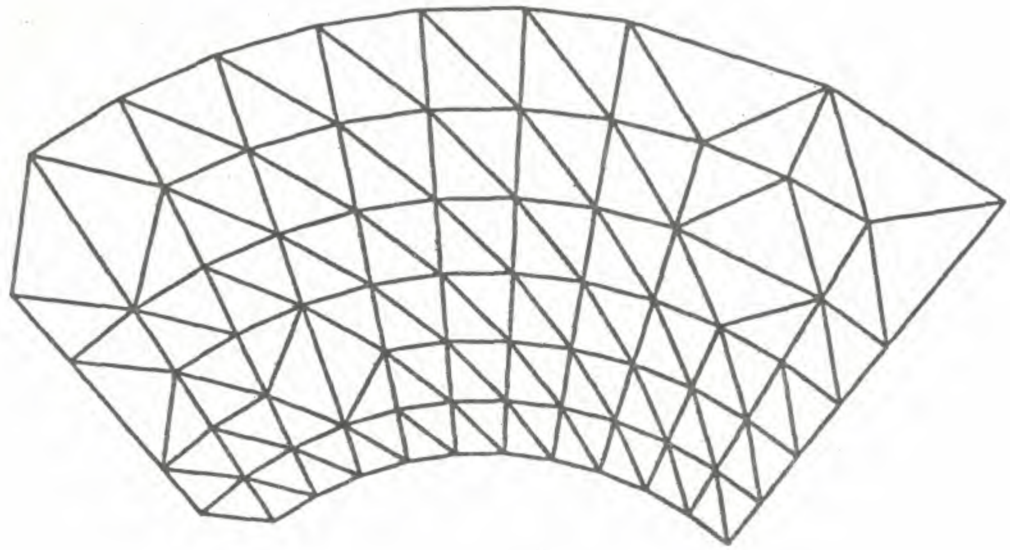

Figure 1. Delaunay triangulation for a portion of a Nelder's wheel. The vertices of the triangulation are the tree locations. The edges define a trees competitors.

Computational efficiency is another important criterion in the choice of a triangulation to create APA. Researchers in the fields of computational geometry and computer science have developed efficient algorithms and programs to create DT's and GT's. The GT can be generated in $O(N^2 \log N)$ run time in the worst case requiring $0(N^2)$ memory (Gilbert 1976). The DT can be created in $O(N \log N)$ run time consistently and only requires O(N) memory (Lee 1980). The benchmark for evaluating the efficiency of these algorithms is O(NlogN), the optimal time for creating any triangulation (Shamos and Hoey 1975). The GT is a better heuristic to the MWT than the DT but is not as efficient for the calculation of APA.

The triangulation networks mentioned are models of first-order neighbors (FON), which are those trees identified as competitors by these triangulations due to their angle and euclidean distance relative to adjoining trees. The FON model assumes that distance and ability to compete are equivalent. This assumption is reasonable for plantations with fairly uniform growth. Heterogeneity in crown class and size could invalidate the use of FON as a model for the identification of competitors, ie., distance from a subject tree may not relate to the choice of the correct competitors. In situations in which the assumption of FON may be weak or incorrect, the triangle network may be used to identify an extended list of possible competitors, including second-order neighbors (SON). A second-order neighbor is the first-order neighbor of a first-order neighbor of a subject tree. Figure 1 is used to illustrate. Number the spokes from 1 to 12 (left to right), number the rows 1 to 7 (top to bottom). The notation (2:1) refers to the top tree on the second spoke from the left. Trees on spoke 7 and rows 3 and 4 are denoted (7:3,4). Tree (7:4) is the subject tree. Its FON's are trees (6:3,4), (7:3,5), and (8:4,5). Its SON's are trees (5:2,3,4), (6:2,5), (7:2,6), (8:3,6), and (9:4,5,6). The recursive nature of this definition makes the identification of any-order neighbor possible by use of the triangle network. The list of FON and SON should be adequate to identify the true competitors.

POLYGONS

APA as defined by Brown is the Voronoi polygon (VP). The VP can be created by calculating the intersections of the perpendicular bisectors of the edges in the DT. The VP was first applied by Thiessen and Alter (1911) for obtaining weighted estimates of rainfall from weather stations in a region. The primary property of the VP which made it useful to Thiessen and for modeling APA is that any point falling within the polygon is closer to the point for which the polygon was created than any other point, a reasonable heuristic for the zone of influence. Another aesthetically pleasing property of the VP is that all polygons have a convex hull, as do most tree crowns (figure 2).

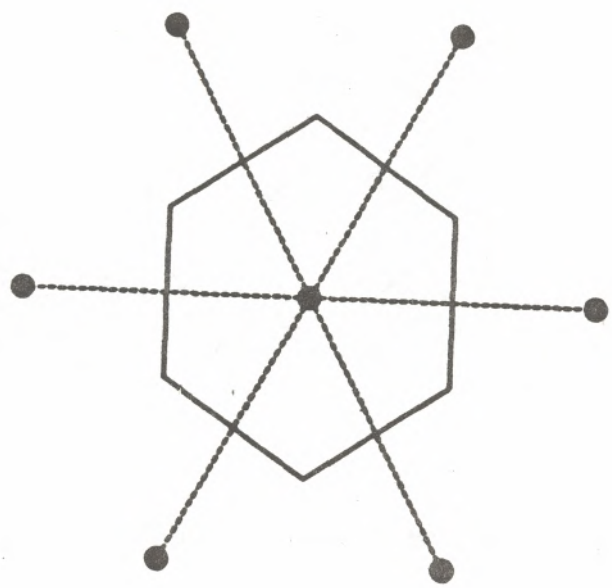

Figure 2. A Voronoi polygon for a diagonal spacing of 10 by 6.

In the same manner that the geometric model must be able to identify which trees can influence others, it must also be able to represent how strongly one tree influences another. The VP assumes that all trees, regardless of size, are equal in their ability to influence another tree. To avoid this dubious assumption, the model should be able to reflect the varying amount of influence exerted by different size trees. Weighting can be used to represent the influence one tree has on another. When these polygons are weighted, "gaps" and "overlaps" will appear between polygons. The idea of areas within a stand not being utilized is intuitively appealing; whether these unassigned and overassigned areas created by unconstrained weighting have any biological relevance has not been determined.

The sampling method developed by Fraser (1977) used a novel method of creating the vertices of the polygons. This method guarantees that there are no gaps or overlaps no matter which weights are used. The weighted midpoints of the edges extending from a subject tree are calculated. The weighted midpoints are always on the hull of the polygon. A vertex between each adjoining edge of a subject is created by calculating the intersection of a line segment drawn from the weighted midpoint of the first edge to the neighbor defined by the adjoining edge and the line segment drawn from the weighted midpoint of the adjoining tree to the neighbor identified by the first edge.[4] In contrast to the VP, Fraser's polygons (FP) do not always contain the area closest to the subject, even when the weights are equal, and are not convex (figure 3). The properties of these polygons will be the same, no matter which triangulation is used as a basis.

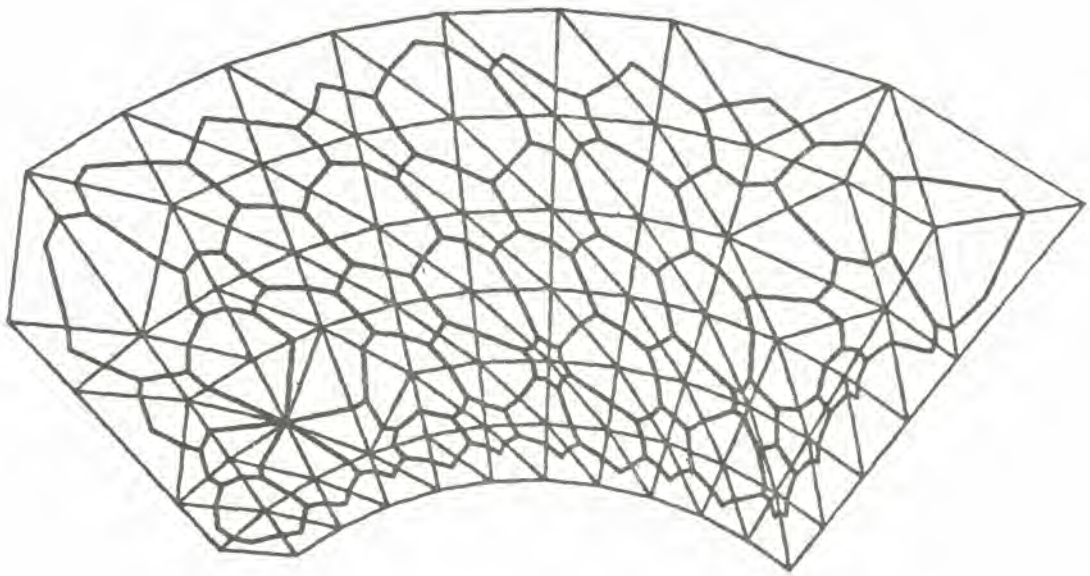

Figure 3. The same Delaunay triangulation as in figure I overlayed by Fraser polygons weighted by dbh^2.

Computing the VP and FP is a very simple and efficient task. Programs developed by Green and Sibson (1977) and Lee (1980) create the VP. The DT is, in fact, most efficently computed by first creating the VP. The Algorithm by Gilbert (1976) can be used to create the GT, though much less efficiently than the DT, and only a few simple procedures need be added to obtain the FP from either triangulation.

FLAWS AND A SOLUTION

There are several flaws and difficulties in creating APA using these geometric models. One difficulty is encountered by the use of FON and SON. The list is not the list of competitors for each subject tree but rather a list of candidates. Competitors must be selected by some criteria from this list. Secondly, the weighted

[4] See Fraser (1977) for further clarification.

or unweighted FP and unweighted VP will tessellate the entire stand without leaving gaps. The resulting APA can be excessive given the size of the tree. The weighted VP has the reverse flaw of causing biologically unjustified gaps and overlaps when weighted. One final difficulty is in creating the APA for border trees on the convex hull of the triangulation. To generate APA with the methods described requires that there not be a region of 180 degrees surrounding the subject tree void of competitors, the definition of a border tree.

To solve these problems, a theoretical limit of influence can be used. The limit defines which FON and SON candidates are capable of interacting by the intersection of their limits. The limit will also correct for gaps, overlaps, and excessive APA and will allow for the creation of APA for border trees. The selection of a limit should have strong biological justification due to its effect on the size and shape of the APA of a tree. The author has used the relationship of crown radius to dbh for open grown trees as a theoretical limit with biological justification. This relationship is species dependent (figure 4).

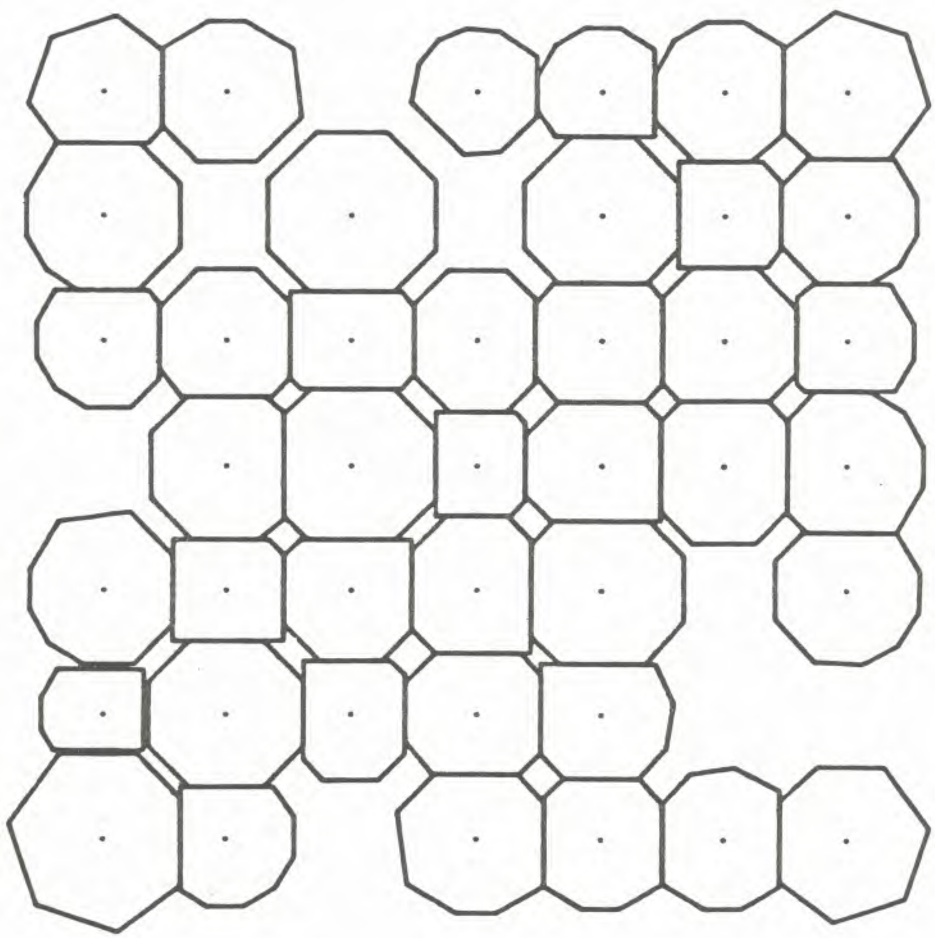

Figure 4. Weighted Voroni polygons with crown radius versus dbh constraint for a spacing of 6 by 6 for a plantation of loblolly pine. The weight is dbh2.

SUMMARY

Several methods of creating APA's and their properties were introduced. The methods used to create these polygons are demonstrated to be computationally efficient and flexible. The APA generated for a tree is shown to have many possible forms based on the assumptions of the spatial relationships of the trees in a stand.

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